

Accelerating black holes, spin-3/2 fields and C-metric

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ABSTRACT: We consider spin-3/2 particles on the background of general accelerating black holes and C -metric. The Rarita-Schwinger equations of spin-3/2 particles are analyzed on these backgrounds. The emission and absorption probabilities of the spin-3/2 particles on these spacetimes are calculated. These backgrounds which we analyze contain both black hole horizon and acceleration horizon, and have general electric and magnetic charges, rotation, and acceleration parameter. The properties of the spin-3/2 field near the acceleration horizon are also computed.

1. Introduction

In this paper, we consider spin-3/2 particles on the spacetime of accelerating black holes. The accelerating black hole is a black hole with the source that has an acceleration [1, 2]. These black holes have an acceleration parameter and can also have charge and rotation. In the coordinate system convenient for a boost-rotation symmetry, the extended space can describe a pair of black holes accelerating in opposite directions [1, 2, 3, 4, 5]. This space is the C -metric, and it can also include charge and rotation [1, 2].

The spin-3/2 particles can be described by the Rarita-Schwinger equations [6]. The spin-3/2 field can appear as effective fields of spin-3/2 baryons, for example hadronic resonances, in effective field theories of interacting hadrons [7, 8]. It can also appear as gravitinos in supergravities in which there is fermionic gauge invariance, for example [9, 10], and their dimensional reductions, for example [9]. The spin-3/2 particle is also of interest for phenomenological models beyond the standard model.

On the other hand, quantum mechanical effects of particles on the background of general relativity can give rise to many interesting phenomena such as the evaporation of black holes via Hawking radiations [11, 12]. These radiations have also been investigated as quantum tunneling of particles from the black hole horizons, for example [13, 14]. Many black holes, such as the Reissner-Nordström and Kerr-Newman black holes, have been studied for these radiations [15-25]. The wave equations of spin-1/2 and spin-0 particles, with or without charge, on three-dimensional black holes such as the charged BTZ spacetimes [26, 27, 28, 29], have also been analyzed [30], and the emission and absorption probability of these particles, incorporating WKB approximation and spacetime symmetries were investigated. The BTZ spacetimes can also appear from the near horizon geometry of higher dimensional black holes in appropriate limits, and can be embedded in supergravity and superstring theory, for example [31, 32]. In a related approach, it has also been shown that the propagation of waves described by the Dirac equation of spin-1/2 fermions in Kerr-Newman spacetime decays in time [33, 34], and the probability that the spin-1/2 particle escapes to infinity was also computed [33]. Fermions with various spins in a curved spacetime have also been studied [21].

In this paper we study the spin-3/2 particles in accelerating black hole spacetimes. We first solve a set of Rarita-Schwinger equations of neutral spin-3/2 particles in the background of accelerating black holes with charge and rotation. An overall phase factor of the wavefunction for the spin-3/2 particles with given energy and angular momentum can be evaluated. The probability of emission and absorption of

spin-3/2 particles across the event horizons are computed. After that, we consider charged spin-3/2 particles on the background of charged accelerating black holes, and a similar analysis are performed. We then analyze the wavefunctions of the spin-3/2 particles near the acceleration horizon, where the emission and absorption probability of the spin-3/2 particles are also computed.

This paper is organized as follows. After an brief introduction of the accelerating charged and rotating black holes in the next Section 2, we discuss spin-3/2 particles on the accelerating black holes in Section 3. After this, charged spin-3/2 particles on charged accelerating black holes are discussed in Section 4. In Section 5 we discuss the acceleration horizon and the emissions of spin-3/2 particles through the acceleration horizon and the temperature associated with it. Finally in Section 6 we make brief conclusions of our paper with some discussion.

2. Accelerating and charged black holes

We consider a family of spacetimes which include an acceleration parameter [1, 2]. It contains the well known spacetimes like Schwarzschild, Reissner-Nordström, Kerr, Kerr-Newman black holes, and many others as its special cases. It also includes accelerating and rotating black holes with zero cosmological constant. Nonzero cosmological constant can also be introduced [1, 2]. The metric for these black holes in spherical polar coordinates (t, r, θ, ϕ) can be written as [1, 2]

$$ds^2 = \frac{1}{\Omega^2} \left\{ -\left(\frac{Q}{\rho^2} - \frac{a^2 P \sin^2 \theta}{\rho^2}\right) dt^2 + \frac{\rho^2}{Q} dr^2 + \frac{\rho^2}{P} d\theta^2 \right. \\ \left. + \left(\frac{P(r^2 + a^2)^2 \sin^2 \theta}{\rho^2} - \frac{Q a^2 \sin^4 \theta}{\rho^2}\right) d\phi^2 \right\} - \frac{2a \sin^2 \theta (P(r^2 + a^2) - Q) dt d\phi}{\rho^2 \Omega^2}, \quad (2.1)$$

in which

$$\Omega = 1 - \alpha r \cos \theta, \quad (2.2)$$

$$\rho^2 = r^2 + a^2 \cos^2 \theta, \quad (2.3)$$

$$P = 1 - 2\alpha M \cos \theta + [\alpha^2 (e^2 + g^2 + a^2)] \cos^2 \theta, \quad (2.4)$$

$$Q = [(a^2 + e^2 + g^2) - 2Mr + r^2] (1 - \alpha^2 r^2). \quad (2.5)$$

Here M is the mass of the black hole, e and g are its electric and magnetic charges, a is the rotation, and α is the acceleration of the black hole. In Eq. (2.1), rearranging

the terms, we get

$$ds^2 = -f(r, \theta)dt^2 + \frac{dr^2}{v(r, \theta)} + \Xi(r, \theta)d\theta^2 + K(r, \theta)d\phi^2 - 2\Phi(r, \theta)dtd\phi, \quad (2.6)$$

where $f(r, \theta)$, $v(r, \theta)$, $\Xi(r, \theta)$, $K(r, \theta)$ and $\Phi(r, \theta)$ are defined below

$$f(r, \theta) = \frac{1}{\Omega^2} \left(\frac{Q - a^2 P \sin^2 \theta}{\rho^2} \right), \quad (2.7)$$

$$v(r, \theta) = \frac{Q\Omega^2}{\rho^2}, \quad (2.8)$$

$$\Xi(r, \theta) = \frac{\rho^2}{P\Omega^2}, \quad (2.9)$$

$$K(r, \theta) = \left(\frac{\sin^2 \theta [P(r^2 + a^2)^2 - Qa^2 \sin^2 \theta]}{\rho^2 \Omega^2} \right), \quad (2.10)$$

$$\Phi(r, \theta) = \left(\frac{a \sin^2 \theta [P(r^2 + a^2) - Q]}{\rho^2 \Omega^2} \right). \quad (2.11)$$

The vector potential for these black holes is

$$A = \frac{-er [dt - a \sin^2 \theta d\phi] - g \cos \theta [adt - (r^2 + a^2) d\phi]}{r^2 + a^2 \cos^2 \theta}. \quad (2.12)$$

These solutions can be obtained in Einstein gravity with Maxwell field.

The horizons are obtained by taking $Q = 0$, which gives their locations at

$$r = \frac{1}{\alpha}, \quad \text{and} \quad r_{\pm} = M \pm \sqrt{M^2 - e^2 - g^2 - a^2}. \quad (2.13)$$

Here r_{\pm} represent the outer and inner horizons similar to those of the Kerr-Newman black holes. We are only considering the case that the sign inside the radical is always positive. The other horizon at $r = \frac{1}{\alpha}$ is an acceleration horizon. In our notations we assume $\alpha \geq 0$.

Now we define the function which will be needed later,

$$F(r, \theta) = f(r, \theta) + \frac{\Phi^2(r, \theta)}{K(r, \theta)}. \quad (2.14)$$

Using the values of $f(r, \theta)$, $K(r, \theta)$ and $\Phi(r, \theta)$ from Eqs. (2.7), (2.10) and (2.11) and after simplification we get

$$F(r, \theta) = \frac{PQ\rho^2}{[P(r^2 + a^2)^2 - Qa^2 \sin^2 \theta] \Omega^2}. \quad (2.15)$$

The angular velocity, for the metric (2.1), takes the form

$$\Omega_H = \left. \frac{-g_{t\phi}}{g_{\phi\phi}} \right|_{r=r_+} = \left. \frac{\Phi(r, \theta)}{K(r, \theta)} \right|_{r=r_+}. \quad (2.16)$$

Using the values of $K(r_+, \theta)$ and $\Phi(r_+, \theta)$ from Eqs. (2.10) and (2.11) we get

$$\Omega_H = \frac{a}{r_+^2 + a^2} \quad (2.17)$$

which is evaluated at the outer horizon.

3. Spin-3/2 particles on accelerating black holes

In this section we consider the accelerating charged and rotating black holes. We consider spin-3/2 fields and their physical properties on the background of these black hole spacetimes. The Rarita-Schwinger equation of the spin 3/2 fermion field is of the form [6]

$$i\gamma^\nu(D_\nu)\Psi_\mu - \frac{m}{\hbar}\Psi_\mu = 0, \quad (3.1)$$

where $\Psi_\mu = \Psi_{\mu\sigma}$ is a vector-valued spinor, with a vector index and a spinor index, and m is the mass of the field. The D_ν is the covariant derivative. The first equation (3.1) is the Dirac equation applied to every vector index of $\Psi_{\mu\sigma}$, while there is a second equation [6]

$$\gamma^\mu\Psi_\mu = 0, \quad (3.2)$$

which is a set of additional constraints. There is a supplementary condition $D^\mu\Psi_\mu = 0$, which can be derived from these above two equations [6]. These constraints ensure that $\Psi_{\mu\sigma}$ represents spin-3/2 fermion fields. The field $\Psi_{\mu\sigma}$ describes the spin-3/2 particle and its anti-particle. The curved space γ -matrices are defined as

$$\begin{aligned} \gamma^t &= \sqrt{\frac{(P(r^2 + a^2)^2 - Qa^2 \sin^2 \theta)(\Omega^2)}{PQ\rho^2}}\gamma^0, & \gamma^r &= \sqrt{\frac{Q\Omega^2}{\rho^2}}\gamma^3, & \gamma^\theta &= \sqrt{\frac{P\Omega^2}{\rho^2}}\gamma^1, \\ \gamma^\phi &= \frac{\rho\Omega\gamma^2}{\sin \theta \sqrt{P(r^2 + a^2)^2 - Qa^2 \sin^2 \theta}} + \frac{a(P(r^2 + a^2) - Q)\gamma^0}{\sqrt{F(r, \theta)(P(r^2 + a^2)^2 - Qa^2 \sin^2 \theta)}}, \end{aligned} \quad (3.3)$$

and we choose the basis for the tangent space γ -matrices to be,

$$\gamma^0 = \begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix},$$

with $\{\gamma^\alpha, \gamma^\beta\} = -2\eta^{\alpha\beta}$.

We use the following ansatz for the wave function

$$\Psi_\mu(t, r, \theta, \phi) = \begin{pmatrix} u_\mu^{(1)} \\ u_\mu^{(2)} \\ u_\mu^{(3)} \\ u_\mu^{(4)} \end{pmatrix} \exp \left[\frac{i}{\hbar} I(t, r, \theta, \phi) \right], \quad (3.4)$$

in which $u_\mu^{(1)}, u_\mu^{(2)}, u_\mu^{(3)}, u_\mu^{(4)}$ each are functions of the spacetime coordinates.

The first Rarita-Schwinger equation (3.1) will give an equation which can be solved for the action I independently of the vector components of the wave function. The second Rarita-Schwinger equation (3.2) will give four constraints for these vector components of the wave function independently of the action. Hence, as the action is all we require to find the horizon temperature, equation (3.2) will never have any effect on the action I . This implies that fermions of every spin will emit at the same temperature.

The covariant derivatives are

$$D_\mu = \partial_\mu + \frac{1}{8} \omega_{\mu\alpha\beta} [\gamma^\alpha, \gamma^\beta], \quad (3.5)$$

where $[\gamma^\alpha, \gamma^\beta]$ satisfies the commutative relations

$$[\gamma^\alpha, \gamma^\beta] = -[\gamma^\beta, \gamma^\alpha], \quad \text{if } \alpha \neq \beta; \quad [\gamma^\alpha, \gamma^\beta] = 0, \quad \text{if } \alpha = \beta. \quad (3.6)$$

By using Eq. (3.6) the Rarita-Schwinger equation takes the form

$$(i\gamma^t \partial_t + i\gamma^r \partial_r + i\gamma^\theta \partial_\theta + i\gamma^\phi \partial_\phi) \Psi_\mu - \frac{m}{\hbar} \Psi_\mu = 0. \quad (3.7)$$

Now, we substitute the above ansatz (3.4) of the wave function into Eq. (3.7) and compute it term by term. We divide by the exponential term and neglect the terms

with higher orders in \hbar . We obtain the following four equations, for $\mu = 0, \dots, 3$,

$$0 = \left(-\frac{1}{\sqrt{F(r, \theta)}} \partial_t I - \sqrt{v(r, \theta)} \partial_r I - \frac{\Phi(r, \theta)}{K(r, \theta) \sqrt{F(r, \theta)}} \partial_\phi I \right) u_\mu^{(3)} + \left(-\frac{1}{\sqrt{\Xi(r, \theta)}} \partial_\theta I + \frac{i}{\sqrt{K(r, \theta)}} \partial_\phi I \right) u_\mu^{(4)} - u_\mu^{(1)} m, \quad (3.8)$$

$$0 = \left(-\frac{1}{\sqrt{F(r, \theta)}} \partial_t I + \sqrt{v(r, \theta)} \partial_r I - \frac{\Phi(r, \theta)}{K(r, \theta) \sqrt{F(r, \theta)}} \partial_\phi I \right) u_\mu^{(4)} - \left(\frac{1}{\sqrt{\Xi(r, \theta)}} \partial_\theta I + \frac{i}{\sqrt{K(r, \theta)}} \partial_\phi I \right) u_\mu^{(3)} - u_\mu^{(2)} m, \quad (3.9)$$

$$0 = \left(\frac{1}{\sqrt{F(r, \theta)}} \partial_t I - \sqrt{v(r, \theta)} \partial_r I + \frac{\Phi(r, \theta)}{K(r, \theta) \sqrt{F(r, \theta)}} \partial_\phi I \right) u_\mu^{(1)} + \left(-\frac{1}{\sqrt{\Xi(r, \theta)}} \partial_\theta I + \frac{i}{\sqrt{K(r, \theta)}} \partial_\phi I \right) u_\mu^{(2)} + u_\mu^{(3)} m, \quad (3.10)$$

$$0 = \left(\frac{1}{\sqrt{F(r, \theta)}} \partial_t I + \sqrt{v(r, \theta)} \partial_r I + \frac{\Phi(r, \theta)}{K(r, \theta) \sqrt{F(r, \theta)}} \partial_\phi I \right) u_\mu^{(2)} - \left(\frac{1}{\sqrt{\Xi(r, \theta)}} \partial_\theta I + \frac{i}{\sqrt{K(r, \theta)}} \partial_\phi I \right) u_\mu^{(1)} + u_\mu^{(4)} m. \quad (3.11)$$

The second set of Rarita-Schwinger equation (3.2) yields

$$0 = \frac{u_t^{(3)}}{\sqrt{F}} + \sqrt{v} u_r^{(3)} + \frac{u_\theta^{(4)}}{\sqrt{\Xi}} + \frac{\Phi u_\phi^{(3)}}{K \sqrt{F}} - \frac{i u_\phi^{(4)}}{\sqrt{K}}, \quad (3.12)$$

$$0 = \frac{u_t^{(4)}}{\sqrt{F}} - \sqrt{v} u_r^{(4)} + \frac{u_\theta^{(3)}}{\sqrt{\Xi}} + \frac{\Phi u_\phi^{(4)}}{K \sqrt{F}} + \frac{i u_\phi^{(3)}}{\sqrt{K}}, \quad (3.13)$$

$$0 = \frac{-u_t^{(1)}}{\sqrt{F}} + \sqrt{v} u_r^{(1)} + \frac{u_\theta^{(2)}}{\sqrt{\Xi}} - \frac{\Phi u_\phi^{(1)}}{K \sqrt{F}} - \frac{i u_\phi^{(2)}}{\sqrt{K}}, \quad (3.14)$$

$$0 = \frac{-u_t^{(2)}}{\sqrt{F}} - \sqrt{v} u_r^{(2)} + \frac{u_\theta^{(1)}}{\sqrt{\Xi}} - \frac{\Phi u_\phi^{(2)}}{K \sqrt{F}} + \frac{i u_\phi^{(1)}}{\sqrt{K}}. \quad (3.15)$$

These will give us additional relation between the various vector components of the wave function, but will not influence the common phase factor given by I in Eq. (3.4), so these are not important here as the solution for the action will be independent of these relations.

Taking into account the symmetries of the spacetime at hand, we employ the ansatz for the action as

$$I = -Et + J\phi + W(r, \theta). \quad (3.16)$$

Here E and J denote the energy and angular momentum of the radiated particle. We denote $\frac{\partial W}{\partial r}$ as W_r for simplicity. Inserting this into the above four equations, we get

$$0 = \left(\frac{E}{\sqrt{F(r, \theta)}} - \sqrt{v(r, \theta)} W_r - \frac{J\Phi(r, \theta)}{K(r, \theta)\sqrt{F(r, \theta)}} \right) u_\mu^{(3)} + \left(-\frac{1}{\sqrt{\Xi(r, \theta)}} W_\theta + \frac{iJ}{\sqrt{K(r, \theta)}} \right) u_\mu^{(4)} - u_\mu^{(1)} m, \quad (3.17)$$

$$0 = \left(\frac{E}{\sqrt{F(r, \theta)}} + \sqrt{v(r, \theta)} W_r - \frac{J\Phi(r, \theta)}{K(r, \theta)\sqrt{F(r, \theta)}} \right) u_\mu^{(4)} - \left(\frac{1}{\sqrt{\Xi(r, \theta)}} W_\theta + \frac{iJ}{\sqrt{K(r, \theta)}} \right) u_\mu^{(3)} - u_\mu^{(2)} m, \quad (3.18)$$

$$0 = \left(-\frac{E}{\sqrt{F(r, \theta)}} - \sqrt{v(r, \theta)} W_r + \frac{J\Phi(r, \theta)}{K(r, \theta)\sqrt{F(r, \theta)}} \right) u_\mu^{(1)} + \left(-\frac{1}{\sqrt{\Xi(r, \theta)}} W_\theta + \frac{iJ}{\sqrt{K(r, \theta)}} \right) u_\mu^{(2)} + u_\mu^{(3)} m, \quad (3.19)$$

$$0 = \left(-\frac{E}{\sqrt{F(r, \theta)}} + \sqrt{v(r, \theta)} W_r + \frac{J\Phi(r, \theta)}{K(r, \theta)\sqrt{F(r, \theta)}} \right) u_\mu^{(2)} - \left(\frac{1}{\sqrt{\Xi(r, \theta)}} W_\theta + \frac{iJ}{\sqrt{K(r, \theta)}} \right) u_\mu^{(1)} + u_\mu^{(4)} m. \quad (3.20)$$

Expanding Eq. (2.8) in Taylor's series and neglecting the higher powers we get

$$v(r, \theta) = v(r_+, \theta) + (r - r_+) v_r(r_+, \theta), \quad (3.21)$$

where we denote $\frac{\partial v}{\partial r}$ as v_r , and similarly for other functions. Note that at the horizon $v(r_+, \theta) = 0$, and its derivative at the horizon is $v_r(r_+, \theta)$, so it becomes

$$v(r, \theta) = (r - r_+) \left(\frac{2(r_+ - M)(1 - \alpha^2 r_+^2) \Omega^2(r_+, \theta)}{(r_+^2 + a^2 \cos^2 \theta)} \right). \quad (3.22)$$

Similarly, we expand $F(r, \theta) = F(r_+, \theta) + (r - r_+) F_r(r_+, \theta)$, and noting that at the horizon $F(r_+, \theta) = 0$, Eq. (2.14) becomes

$$F(r, \theta) = (r - r_+) \left(\frac{2(r_+^2 + a^2 \cos^2 \theta)(r_+ - M)(1 - \alpha^2 r_+^2)}{(r_+^2 + a^2)^2 \Omega^2(r_+, \theta)} \right). \quad (3.23)$$

Expanding near the black hole horizon, from Eqs. (3.12) to (3.15), we obtain four equations which have no effects on the solution of the action. Now expanding Eqs.

(3.17) to (3.20) near the black hole horizon and using Eqs. (3.22) and (3.23), we get the first Rarita-Schwinger equation in the simplified form,

$$\begin{bmatrix} -m & 0 & \varepsilon - W_r \sqrt{(r - r_+) v_r} & s - \frac{W_\theta}{\sqrt{\Xi}} \\ 0 & -m & -s - \frac{W_\theta}{\sqrt{\Xi}} & \varepsilon + W_r \sqrt{(r - r_+) v_r} \\ \varepsilon + W_r \sqrt{(r - r_+) v_r} & -s + \frac{W_\theta}{\sqrt{\Xi}} & -m & 0 \\ s + \frac{W_\theta}{\sqrt{\Xi}} & \varepsilon - W_r \sqrt{(r - r_+) v_r} & 0 & -m \end{bmatrix} \begin{bmatrix} u_\mu^{(1)} \\ u_\mu^{(2)} \\ u_\mu^{(3)} \\ u_\mu^{(4)} \end{bmatrix} = 0, \quad (3.24)$$

where we define

$$\varepsilon = \frac{E}{\sqrt{(r - r_+) F_r(r_+, \theta)}} - \frac{J \Phi}{K \sqrt{(r - r_+) F_r(r_+, \theta)}}, \quad (3.25)$$

$$s = i \frac{J}{\sqrt{K}}. \quad (3.26)$$

We make a Left-Upper Decomposition of the matrix and this gives,

$$(\varepsilon + W_r \sqrt{(r - r_+) v_r})(\varepsilon - W_r \sqrt{(r - r_+) v_r}) - (s + \frac{W_\theta}{\sqrt{\Xi}})(-s + \frac{W_\theta}{\sqrt{\Xi}}) - m^2 = 0. \quad (3.27)$$

It can be seen that we can take $\theta = \theta_0$, where θ_0 is constant of motion, and then W_θ can be solved to be a constant. Then expanding Eq. (3.27) near the horizon $r = r_+$ and solving for W_r , we see that only ε contributes, as the absolute value of W_r increases quickly to infinity near the horizon while the other terms do not contribute in the solution. Near the horizon we see that $W(r, \theta)$ can be written as $W(r) + \xi(\theta)$, where ξ is complex and has no r dependence. We see that the W_r has two solutions, $\partial_r W_+$ and $\partial_r W_-$, corresponding to outgoing and incoming modes respectively,

$$\partial_r W_\pm = \pm \frac{(E - \Omega_H J)}{\sqrt{(r - r_+) F_r(r_+, \theta)} \sqrt{(r - r_+) v_r(r_+, \theta)}}. \quad (3.28)$$

Substituting the values of $F_r(r_+, \theta)$ and $v_r(r_+, \theta)$, the above equations becomes

$$\partial_r W_\pm = \pm \frac{(E - \Omega_H J)(r_+^2 + a^2)}{2(r - r_+)(r_+ - M)(1 - \alpha^2 r_+^2)}. \quad (3.29)$$

The $\partial_r W_\pm$ has a pole at the horizon r_+ . For finding the value of W we integrate the above result

$$W_\pm = \pm \int \frac{(E - \Omega_H J)(r_+^2 + a^2) dr}{2(r - r_+)(r_+ - M)(1 - \alpha^2 r_+^2)}. \quad (3.30)$$

Integrating the above integrand around the pole $r = r_+$, this gives

$$W_{\pm} = \pm \pi i \frac{(E - \Omega_H J)(r_+^2 + a^2)}{2(r_+ - M)(1 - \alpha^2 r_+^2)} \quad (3.31)$$

and we obtain

$$\text{Im } W_{\pm} = \pm \frac{\pi (E - \Omega_H J)(r_+^2 + a^2)}{2(r_+ - M)(1 - \alpha^2 r_+^2)}. \quad (3.32)$$

So the probabilities of the emission and absorption of the spin-3/2 particles are

$$P_{\text{emission}} \propto \exp[-2 \text{Im } I_+] = \exp[-2(\text{Im } W_+ + \text{Im } \xi)], \quad (3.33)$$

$$P_{\text{absorption}} \propto \exp[-2 \text{Im } I_-] = \exp[-2(\text{Im } W_- + \text{Im } \xi)]. \quad (3.34)$$

Since $\text{Im } W_+ = -\text{Im } W_-$,

$$\Gamma = \frac{P_{\text{emission}}}{P_{\text{absorption}}} = \frac{\exp[-2(\text{Im } W_+)]}{\exp[-2(\text{Im } W_-)]} = \exp[-4 \text{Im } W_+]. \quad (3.35)$$

The resulting tunneling probability is

$$\Gamma = \exp[-2\pi \frac{(E - \Omega_H J)(r_+^2 + a^2)}{(r_+ - M)(1 - \alpha^2 r_+^2)}] = \exp[-\beta(E - \Omega_H J)]. \quad (3.36)$$

Comparing this with $\beta = 1/T_H$ we find that the horizon temperature is given by

$$T_H = \frac{(r_+ - M)(1 - \alpha^2 r_+^2)}{2\pi(r_+^2 + a^2)}, \quad (3.37)$$

where r_+ is given by Eq. (2.13). If we set rotation and acceleration equal to zero, these expressions will give the expressions for the Reissner-Nordström black hole. If we compare it with the black hole without acceleration, we note that the effect of acceleration is that it decreases the temperature.

4. The charged case

In this section we consider charged spin-3/2 particles on the accelerating and rotating charged black hole, with the outer horizon $r_+ = M + \sqrt{M^2 - e^2 - g^2 - a^2}$. The Rarita-Schwinger equation of the spin 3/2 fermion Ψ_μ with charge q is given by

$$i\gamma^\nu \left(D_\nu - \frac{iq}{\hbar} A_\nu \right) \Psi_\mu - \frac{m}{\hbar} \Psi_\mu = 0, \quad (4.1)$$

where q and m is the charge and mass of the particle and A_μ is the vector potential.

Using an ansatz similar to the one in Section 3, the above wave equation takes the form

$$i\gamma^t \partial_t \Psi_\mu + i\gamma^r \partial_r \Psi_\mu + i\gamma^\theta \partial_\theta \Psi_\mu + i\gamma^\phi \partial_\phi \Psi_\mu + \gamma^t \frac{q}{\hbar} A_t \Psi_\mu + \gamma^\phi \frac{q}{\hbar} A_\phi \Psi_\mu - \frac{m}{\hbar} \Psi_\mu = 0. \quad (4.2)$$

Substituting the functions as in Section 2 and using the vector potential A_μ given by Eq. (2.12), after simplification, we obtain four equations,

$$\begin{aligned} 0 = & -\left[\frac{1}{\sqrt{F(r, \theta)}} \partial_t I + \sqrt{v(r, \theta)} \partial_r I + \frac{\Phi(r, \theta)}{K(r, \theta) \sqrt{F(r, \theta)}} \partial_\phi I - \frac{1}{\sqrt{F(r, \theta)}} q A_t \right. \\ & \left. - \frac{\Phi(r, \theta)}{K(r, \theta) \sqrt{F(r, \theta)}} q A_\phi\right] u_\mu^{(3)} + \left[-\frac{1}{\sqrt{\Xi(r, \theta)}} \partial_\theta I + \frac{i}{\sqrt{K(r, \theta)}} \partial_\phi I - \frac{i}{\sqrt{K(r, \theta)}} q A_\phi\right] u_\mu^{(4)} - u_\mu^{(1)} m, \end{aligned} \quad (4.3)$$

$$\begin{aligned} 0 = & -\left[\frac{1}{\sqrt{F(r, \theta)}} \partial_t I - \sqrt{v(r, \theta)} \partial_r I + \frac{\Phi(r, \theta)}{K(r, \theta) \sqrt{F(r, \theta)}} \partial_\phi I - \frac{1}{\sqrt{F(r, \theta)}} q A_t \right. \\ & \left. - \frac{\Phi(r, \theta)}{K(r, \theta) \sqrt{F(r, \theta)}} q A_\phi\right] u_\mu^{(4)} - \left[\frac{1}{\sqrt{\Xi(r, \theta)}} \partial_\theta I + \frac{i}{\sqrt{K(r, \theta)}} \partial_\phi I - \frac{i}{\sqrt{K(r, \theta)}} q A_\phi\right] u_\mu^{(3)} - u_\mu^{(2)} m, \end{aligned} \quad (4.4)$$

$$\begin{aligned} 0 = & \left[\frac{1}{\sqrt{F(r, \theta)}} \partial_t I - \sqrt{v(r, \theta)} \partial_r I + \frac{\Phi(r, \theta)}{K(r, \theta) \sqrt{F(r, \theta)}} \partial_\phi I - \frac{1}{\sqrt{F(r, \theta)}} q A_t \right. \\ & \left. - \frac{\Phi(r, \theta)}{K(r, \theta) \sqrt{F(r, \theta)}} q A_\phi\right] u_\mu^{(1)} + \left[-\frac{1}{\sqrt{\Xi(r, \theta)}} \partial_\theta I + \frac{i}{\sqrt{K(r, \theta)}} \partial_\phi I - \frac{i}{\sqrt{K(r, \theta)}} q A_\phi\right] u_\mu^{(2)} + u_\mu^{(3)} m, \end{aligned} \quad (4.5)$$

$$\begin{aligned} 0 = & \left[\frac{1}{\sqrt{F(r, \theta)}} \partial_t I + \sqrt{v(r, \theta)} \partial_r I + \frac{\Phi(r, \theta)}{K(r, \theta) \sqrt{F(r, \theta)}} \partial_\phi I - \frac{1}{\sqrt{F(r, \theta)}} q A_t \right. \\ & \left. - \frac{\Phi(r, \theta)}{K(r, \theta) \sqrt{F(r, \theta)}} q A_\phi\right] u_\mu^{(2)} - \left[\frac{1}{\sqrt{\Xi(r, \theta)}} \partial_\theta I + \frac{i}{\sqrt{K(r, \theta)}} \partial_\phi I - \frac{i}{\sqrt{K(r, \theta)}} q A_\phi\right] u_\mu^{(1)} + u_\mu^{(4)} m. \end{aligned} \quad (4.6)$$

From Eq. (3.12) to Eq. (3.15), we obtained four equations which have no effects on the solution of the action.

In order to solve these equations (4.3) to (4.6), we again employ an action of the form given in Eq. (3.4) and substitute it in the above equations. Near the horizon

at $r = r_+$, the functions take the following form,

$$\begin{aligned} F(r, \theta) &= (r - r_+) \partial_r F(r_+, \theta) \\ &= (r - r_+) \frac{(r_+^2 + a^2 \cos^2 \theta) (2r_+ - 2M) (1 - \alpha^2 r_+^2)}{(1 - \alpha r_+ \cos \theta)^2 (r_+^2 + a^2)^2}, \end{aligned} \quad (4.7)$$

$$\begin{aligned} v(r, \theta) &= (r - r_+) \partial_r v(r_+, \theta) \\ &= (r - r_+) \frac{(1 - \alpha r_+ \cos \theta)^2 (2r_+ - 2M) (1 - \alpha^2 r_+^2)}{(r_+^2 + a^2 \cos^2 \theta)}, \end{aligned} \quad (4.8)$$

$$\Omega_H = \frac{\Phi(r_+, \theta)}{K(r_+, \theta)} = \frac{a}{(r_+^2 + a^2)}. \quad (4.9)$$

Using these values in Eq. (4.3) to Eq. (4.6) and expanding near the horizon we obtain

$$0 = (\varepsilon - W_r \sqrt{(r - r_+) v_r(r_+, \theta)}) u_\mu^{(3)} + (s - \frac{W_\theta}{\sqrt{\Xi(r_+, \theta)}}) u_\mu^{(4)} - u_\mu^{(1)} m, \quad (4.10)$$

$$0 = (\varepsilon + W_r \sqrt{(r - r_+) v_r(r_+, \theta)}) u_\mu^{(4)} + (-s - \frac{W_\theta}{\sqrt{\Xi(r_+, \theta)}}) u_\mu^{(3)} - u_\mu^{(2)} m, \quad (4.11)$$

$$0 = (\varepsilon + W_r \sqrt{(r - r_+) v_r(r_+, \theta)}) u_\mu^{(1)} - (-s - \frac{W_\theta}{\sqrt{\Xi(r_+, \theta)}}) u_\mu^{(2)} - u_\mu^{(3)} m, \quad (4.12)$$

$$0 = (\varepsilon - W_r \sqrt{(r - r_+) v_r(r_+, \theta)}) u_\mu^{(2)} - (s - \frac{W_\theta}{\sqrt{\Xi(r_+, \theta)}}) u_\mu^{(1)} - u_\mu^{(4)} m, \quad (4.13)$$

where we define

$$\varepsilon = \frac{\left(E - \Omega_H J - \frac{qer_+}{(r_+^2 + a^2)} \right)}{\sqrt{(r - r_+) \partial_r F(r_+, \theta)}}, \quad (4.14)$$

$$s = i \left(\frac{J}{\sqrt{K}} + \frac{qer}{\rho^2 \sqrt{K}} a \sin^2 \theta \right). \quad (4.15)$$

Near the horizon, we also have the functions as following,

$$\Xi(r_+, \theta) = \frac{\rho^2(r_+, \theta)}{P \Omega^2(r_+, \theta)}, \quad (4.16)$$

$$K(r_+, \theta) = \left(\frac{\sin^2 \theta P (r_+^2 + a^2)^2}{\rho^2(r_+, \theta) \Omega^2(r_+, \theta)} \right), \quad (4.17)$$

$$\Phi(r_+, \theta) = \left(\frac{a \sin^2 \theta P (r_+^2 + a^2)}{\rho^2(r_+, \theta) \Omega^2(r_+, \theta)} \right). \quad (4.18)$$

Solving the above equations (4.10) to (4.13), as we discussed in Section 3, finally we get

$$W_{\pm} = \pm \frac{\pi i \left(E - \Omega_H J - \frac{qer_+}{(r_+^2 + a^2)} \right) (r_+^2 + a^2)}{(2r_+ - 2M)(1 - \alpha^2 r_+^2)}, \quad (4.19)$$

and

$$\text{Im } W_{\pm} = \pm \frac{\pi}{2} \frac{\left(E - \Omega_H J - \frac{qer_+}{(r_+^2 + a^2)} \right) (r_+^2 + a^2)}{(r_+ - M)(1 - \alpha^2 r_+^2)}. \quad (4.20)$$

So the tunneling probability becomes

$$\Gamma = \frac{P_{\text{emission}}}{P_{\text{absorption}}} = \exp[-4 \text{Im } W_+]. \quad (4.21)$$

Using the value of $\text{Im } W_+$ in the above equation we obtain

$$\Gamma = \exp\left[-\frac{2\pi(r_+^2 + a^2)}{(r_+ - M)(1 - \alpha^2 r_+^2)} \left(E - \Omega_H J - \frac{qer_+}{(r_+^2 + a^2)} \right)\right] \quad (4.22)$$

$$= \exp[-\beta(E - \Omega_H J - V_H q)], \quad (4.23)$$

which is the Boltzmann factor of the emitted charged particles, including the chemical potential conjugate to the charge q . Comparing this expression with $\beta = 1/T_H$, this gives the horizon temperature as

$$T_H = \frac{(r_+ - M)(1 - \alpha^2 r_+^2)}{2\pi(r_+^2 + a^2)}, \quad (4.24)$$

and r_+ in this case is given by Eq. (2.13). If we put acceleration and rotation equal to zero in formulae (4.22) and (4.24), they reduce to the tunneling probability and the temperature of the Reissner-Nordström black hole. If we set the charges of the background to be zero, they also recover the expressions in Section 3.

5. The acceleration horizon and the charged rotating C -metric

The charged C -metric can give a description of a Reissner-Nordström black hole with constant proper acceleration and with mass M and charge e , for example [1, 5]. In the extended space with the coordinate system adapted to boost-rotation symmetry, the complete space can describe a pair of charged black holes with uniform accelerations in opposite directions, for example [1, 2, 3, 4, 5]. It is an example of boost-rotation

symmetric spacetime, with an axial Killing vector and a boost Killing vector. The spin-3/2 particles can be emitted through the horizons of these backgrounds.

In the metric of these backgrounds (2.1), the range of θ is $\theta \in [0, \pi]$. The function $\Omega = 1 - \alpha r \cos \theta$, has no zero for $\theta \in [\frac{\pi}{2}, \pi]$. Due to the $\frac{1}{\Omega^2}$ factor of the metric, for $\theta \in [0, \frac{\pi}{2}]$, the conformal infinity is located at the zero of the function Ω , which is $r = \frac{1}{\alpha \cos \theta} [1, 2]$.

The horizons of these spacetimes are obtained by setting $Q = 0$ in Eq. (2.5), which have three positive roots. The two roots

$$r_{\pm} = M \pm \sqrt{M^2 - e^2 - g^2 - a^2}, \quad (5.1)$$

represent the outer and inner horizons. We are only considering the case that the sign inside the radical is always positive. The other root is

$$r_a = \frac{1}{\alpha}, \quad (5.2)$$

which is an acceleration horizon related to a boost symmetry, and the subscript a in r_a denotes acceleration.

Now we consider a different solution of the function W for the spin-3/2 particles defined in Eq. (3.16) in Section 3. In Sections 3 and 4, we have the solution where the pole of W_r is at $r = r_+$. Now we look for a different solution where the pole of W_r is at $r = r_a$. We then get the set of equations in the form of Eq. (4.3) to Eq. (4.6). Near the acceleration horizon, both $F(r, \theta)$ and $v(r, \theta)$ have a zero at r_a , so we can expand them as $F(r, \theta) = F(r_a, \theta) + (r - r_a)F_r(r_a, \theta)$ and $v(r, \theta) = v(r_a, \theta) + (r - r_a)v_r(r_a, \theta)$. We then get four equations in the similar form as Eq. (4.10) to Eq. (4.13) in Section 4, except that in this section we expand the functions at a different zero of the function Q . The W_r has two solutions, $\partial_r W_+$ and $\partial_r W_-$, corresponding to outgoing and incoming modes,

$$\partial_r W_{\pm} = \pm \frac{(E - \Omega_a J - \frac{qe\alpha}{1+a^2\alpha^2})}{\sqrt{(r - r_a)F_r} \sqrt{(r - r_a)v_r}} \quad (5.3)$$

$$= \pm \frac{(1 + a^2\alpha^2)(E - \Omega_a J - \frac{qe\alpha}{1+a^2\alpha^2})}{2\alpha(r - r_a)(1 - 2M\alpha + (e^2 + g^2 + a^2)\alpha^2)}, \quad (5.4)$$

where $\Omega_a = \frac{a}{r_a^2 + a^2}$. By integrating the above integrand around the pole at $r = r_a$, we get

$$W_{\pm} = \pm i \frac{\pi(1 + a^2\alpha^2)(E - \Omega_a J - \frac{qe\alpha}{1+a^2\alpha^2})}{2\alpha(1 - 2M\alpha + (e^2 + g^2 + a^2)\alpha^2)}, \quad (5.5)$$

and

$$\Gamma = \exp[-4 \operatorname{Im} W_+] = \exp[-\beta(E - \Omega_a J - \frac{qe\alpha}{1 + a^2\alpha^2})]. \quad (5.6)$$

Comparing this expression with $\beta = 1/T$, we get the temperature of the emitted spin-3/2 particles at the acceleration horizon,

$$T_a = \frac{\alpha(1 - 2M\alpha + (e^2 + g^2 + a^2)\alpha^2)}{2\pi(1 + a^2\alpha^2)}, \quad (5.7)$$

where the subscript a in T_a denotes acceleration.

Now we look at the surface gravity at the horizons. There is a coordinate singularity at the black hole event horizon r_+ and at the acceleration horizon r_a . These can be removed by using Painlevé-type coordinate transformation. We perform the coordinate transformation,

$$dt \rightarrow dt - \sqrt{\frac{1-v}{Fv}} dr, \quad (5.8)$$

so that the Eq. (2.1) becomes,

$$ds^2 = -F(r, \theta)dt^2 + dr^2 + 2F\sqrt{\frac{1-v}{Fv}}drdt + \Xi(r, \theta)d\theta^2 + K(r, \theta)d\bar{\phi}^2, \quad (5.9)$$

where $d\bar{\phi} = d\phi - \frac{\Phi(r, \theta)}{K(r, \theta)}dt$. In this form the surface gravity becomes

$$\kappa|_{r=r_*} = \frac{1}{2} \sqrt{\frac{1-v}{Fv}} v \frac{dF}{dr} \bigg|_{r=r_*}, \quad (5.10)$$

where r_* denotes the locations of the horizons, the black hole horizon r_+ and the acceleration horizon r_a .

Using this formula (5.10) for the C -metric Eq. (5.9), we obtain result at the black hole event horizon,

$$\kappa|_{r=r_+} = \frac{(r_+ - M)(1 - \alpha^2 r_+^2)}{r_+^2 + a^2}, \quad (5.11)$$

where r_+ is the outer horizon in Eq. (5.1). The temperature at the black hole horizon, $T = \kappa/2\pi$, of the C -metric agrees with the computation in Eq. (4.24). By using Eq. (5.10), the surface gravity at the acceleration horizon is,

$$\kappa|_{r=r_a} = \frac{\alpha(1 - 2M\alpha + (e^2 + g^2 + a^2)\alpha^2)}{1 + a^2\alpha^2}. \quad (5.12)$$

The surface gravity at the acceleration horizon of the C -metric thus agrees with the computation of Eq. (5.7), that is $\kappa|_{r=r_a} = 2\pi T_a$.

6. Discussion

We have considered the Rarita-Schwinger equations for spin-3/2 particles on the backgrounds of accelerating black holes with general electric charge, magnetic charge, rotation, and acceleration parameter. We first consider the neutral spin-3/2 particles. The overall phase factor in the vector components of the wavefunctions for the spin-3/2 particle are analyzed and its radial derivative has poles on the locations of the event horizons. By the integration along complex path we evaluated the emission and absorption probabilities of the spin-3/2 particles across the horizons, as well as the temperatures of the horizons.

We also analyzed the charged spin-3/2 particles, on the background of these accelerating black holes, with parameters of electric and magnetic charges and rotation. The spin-3/2 wave equations are analyzed similarly for this case. The Boltzmann factor of the emitted outgoing particles are derived, including the chemical potential conjugate to the charge of the particle.

Because the spacetime here is a black hole with acceleration, there are two types of event horizons. One is the black hole event horizon at the outer horizon radius, and another is the acceleration horizon due to the acceleration of the source. By the integration along complex path we obtained also the emission and absorption probabilities of the spin-3/2 particles across the acceleration horizon.

The properties of the wavefunctions of the spin-3/2 particles near the acceleration horizon are also analyzed in detail. The surface gravity at the acceleration horizon calculated after performing a coordinate transformation, matches with the temperature calculated by the Rarita-Schwinger equations.

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References

- [1] J. B. Griffiths, P. Krtouš and J. Podolský, *Class. Quant. Grav.* **23** (2006) 6745 [gr-qc/0609056].
- [2] J. B. Griffiths and J. Podolský, *Class. Quant. Grav.* **23**, 555 (2006) [gr-qc/0511122].

- [3] J. Bičák, Proc. R. Soc. Lond. A, 1980, vol. **371**, no. 1746, 429.
- [4] V. Pravda and A. Pravdová, Class. Quant. Grav. **18** (2001) 1205 [gr-qc/0010051].
- [5] F. H. J. Cornish, W. J. Uttley, Gen. Rel. Grav. **27**, 735 (1995).
- [6] W. Rarita and J. Schwinger, Phys. Rev. **60** (1941) 61.
- [7] M. Benmerrouche, R. M. Davidson and N. C. Mukhopadhyay, Phys. Rev. C **39**, 2339 (1989).
- [8] T. R. Hemmert, B. R. Holstein and J. Kambor, J. Phys. G **24** (1998) 1831 [hep-ph/9712496].
- [9] S. D. Rindani and M. Sivakumar, J. Phys. G **12** (1986) 1335.
- [10] S. Ferrara, M. Porrati and V. L. Telegdi, Phys. Rev. D **46** (1992) 3529.
- [11] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975).
- [12] G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2738 (1977).
- [13] P. Kraus and F. Wilczek, Mod. Phys. Lett. A **09**, 3713 (1994) [gr-qc/9406042].
- [14] M. K. Parikh and F. Wilczek, Phys. Rev. Lett. **85**, 5042 (2000) [hep-th/9907001].
- [15] S. Shankaranarayanan, K. Srinivasan and T. Padmanabhan, Mod. Phys. Lett. A **16**, 571 (2001) [gr-qc/0007022].
- [16] K. Srinivasan and T. Padmanabhan, Phys. Rev. D **60**, 024007 (1999) [gr-qc/9812028].
- [17] T. Damour and R. Ruffini, Phys. Rev. D **14**, 332 (1976).
- [18] R. Kerner and R. B. Mann, Class. Quant. Grav. **25**, 095014 (2008) [arXiv:0710.0612 [hep-th]].
- [19] U. A. Gillani, M. Rehman and K. Saifullah, JCAP **1106**, 016 (2011) [arXiv:1102.0029 [hep-th]].
- [20] H. Gohar and K. Saifullah, Astrophys. Space Sci. **343** (2013) 181 [arXiv:1109.5836 [hep-th]].
- [21] A. Yale and R. B. Mann, Phys. Lett. B **673** (2009) 168 [arXiv:0808.2820 [gr-qc]].
- [22] R. Li and J.-R. Ren, Phys. Lett. B **661**, 370 (2008) [arXiv:0802.3954 [gr-qc]].
- [23] A. J. M. Medved, Class. Quant. Grav. **19**, 589 (2002) [hep-th/0110289].

- [24] U. A. Gillani and K. Saifullah, Phys. Lett. B **699** (2011) 15 [arXiv:1010.6106 [hep-th]].
- [25] V. Akhmedova, T. Pilling, A. de Gill and D. Singleton, Phys. Lett. B **666**, 269 (2008) [arXiv:0804.2289 [hep-th]].
- [26] M. Bañados, C. Teitelboim and J. Zanelli, Phys. Rev. Lett. **69**, 1849 (1992) [hep-th/9204099].
- [27] M. Bañados, M. Henneaux, C. Teitelboim and J. Zanelli, Phys. Rev. D **48**, 1506 (1993) [gr-qc/9302012].
- [28] C. Martínez, C. Teitelboim and J. Zanelli, Phys. Rev. D **61**, 104013 (2000) [hep-th/9912259].
- [29] S. Carlip and J. Gegenberg, Phys. Rev. D **44**, 424 (1991).
- [30] A. Ejaz, H. Gohar, H. Lin, K. Saifullah and S. -T. Yau, Phys. Lett. B **726**, 827 (2013) [arXiv:1306.6380 [hep-th]].
- [31] V. Balasubramanian, J. de Boer, V. Jejjala and J. Simon, JHEP **0805**, 067 (2008) [arXiv:0707.3601 [hep-th]].
- [32] R. Fareghbal, C. N. Gowdigere, A. E. Mosaffa and M. M. Sheikh-Jabbari, JHEP **0808**, 070 (2008) [arXiv:0801.4457 [hep-th]].
- [33] F. Finster, N. Kamran, J. Smoller, S. -T. Yau, Comm. Math. Phys. **230** (2002), no. 2, 201–244 [gr-qc/0107094].
- [34] F. Finster, N. Kamran, J. Smoller and S. -T. Yau, Adv. Theor. Math. Phys. **7** (2003) 25 [gr-qc/0005088].